ASSIGNMENT 2

Submitted By

Zajiba Sadia Islam

Student ID: 501279357

Group-09

Submitted for

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Course: Stats for the Health Sciences

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**Probability Problems**

**1.**

A table of numbers and text

Description automatically generated

**Solve**:

From the provided life table, we can see that the survival probability for females (wives) at age 70 is given by lx​, which is 80,145 out of 100,000.

For males (husbands), the survival probability at age 70 is also given by lx, which is 80,145 out of 100,000.

The probability that either the wife or the husband, but not both, will be alive at age 70:

P(Either wife or husband alive at age 70)=P(Wife alive at age 70)+P(Husband alive at age 70)−P(Both alive at age 70)

= l70,wife/100,000+ l70,husband/100,000− {(l70,wife × l70,husband)/(100,000×100,000)}

=80145/100,000+80145/100,000−{(80145×80145)/(100,000×100,000)} ​

=160290/100,000−6418481025/10,000,000,000

=1.6029%−0.0641%

=1.5387%

So, the probability that either the wife or the husband, but not both, will be alive at age 70 is approximately 1.5387%

**2. (23a)** The computers of six faculty members in a certain department are to be replaced. Two of the faculty members have selected laptop machines and the other four have chosen desktop machines. Suppose that only two of the setups can be done on a particular day, and the two computers to be set up are randomly selected from the six (implying 15 equally likely outcomes; if the computers are numbered 1, 2…, 6, then one outcome consists of computers 1 and 2, another consists of computers 1 and 3, and so on). What is the probability that both selected setups are for laptop computers?

**Solve:**

Total number of ways to select 2 setups=6!/ 2!(6−2)! ​=(6×5)/(2\*1)​=15

Since 2 faculty members have selected laptops, there are 2C2=1 way to choose 2 setups from those 2 laptops.

Therefore, the probability of selecting 2 laptop setups is:

P(Both setups are laptops)=Number of ways to select 2 laptop setups/Total number of ways to select 2 setups

P(Both setups are laptops)=1/15

So, the probability that both selected setups are for laptop computers is 1/15.

**3. (51a)** According to a July 31, 2013, posting on cnn.com subsequent to the death of a child who bit into a peanut, a 2010 study in the journal Pediatrics found that 8% of children younger than 18 in the United States have at least one food allergy. Among those with food allergies, about 39% had a history of severe reaction. If a child younger than 18 is randomly selected, what is the probability that he or she has at least one food allergy and a history of severe reaction?

**Solve:**

Let: A be the event that a child younger than 18 has at least one food allergy.

B be the event that a child younger than 18 with a food allergy has a history of severe reaction.

* *P*(*A*), the probability that a child younger than 18 has at least one food allergy, is 8% or 0.08.
* *P*(*B*∣*A*), the conditional probability that a child with a food allergy has a history of severe reaction, is 39% or 0.39.

*P*(*A*∩*B*), the probability that a child younger than 18 has at least one food allergy and a history of severe reaction.

Using conditional probability, we obtain:

*P*(*B*∣*A*)= *P*(*A*∩*B*)​/ *P*(*A*)

Or, *P*(*A*∩*B*)=*P*(*B*∣*A*)×*P*(*A*)

Substituting the given probabilities we get:

*P*(*A*∩*B*)=0.39×0.08=0.0312

So, the probability that a randomly selected child younger than 18 has at least one food allergy and a history of severe reaction is 0.0312, or 3.12%.

Top of Form

**Probability Distributions and Sampling Problems**

1. **(11c)** According to the National Health Survey, 9.8% of the population of 18- to 24-year-olds in the United States are left-handed. What is the probability that exactly three of the ten persons are left-handed?

**Solve:**

Using the binomial probability formula:

*P*(*X*=*k*)=××(

Where:

* *P*(*X*=*k*) is the probability of getting exactly *k* successes (in this case, exactly 3 left-handed persons).
* *n* is the total number of trials (in this case, 10 persons).
* *p* is the probability of success on each trial (in this case, the probability of a person being left-handed, which is 9.8% or 0.098).
* (1−*p*) is the probability of failure on each trial (in this case, the probability of a person not being left-handed, which is 1−0.098=0.9021−0.098=0.902).

By plugging in the values:

*P*(*X*=3)=××(

*P*(*X*=3)=120×0.0009113648×0.323841226

*P*(*X*=3)≈0.03555

So, the probability that exactly three of the ten persons are left-handed is approximately 0.03555, or about 3.555%.

1. (17a) Consider the standard normal distribution with mean μ= 0 and standard deviation σ= 1. What is the probability that an outcome z is greater than 2.60?

**Solve**:

Since the standard normal distribution is symmetric about the mean (*μ*=0), we know that the area to the left of the mean (*z*=0) is 0.5. Therefore, we only need to find the area to the right of *z*=2.60.

For a z-score of 2.60, the area to the left is approximately 0.9953. Therefore, the area to the right is:

*P*(*z*>2.60)=1−0.9953=0.0047

So, the probability that an outcome *z* is greater than 2.60 is approximately 0.0047, or 0.47%.

1. (7.2-10b) Fawns between 1 and 5 months old in Mesa Verde National Park have a body weight that is approximately normally distributed with mean μ = 27.2 kilograms and standard deviation σ = 4.3 kilograms (based on information from The Mule Deer of Mesa Verde National Park, by G. W. Mierau and J. L. Schmidt, Mesa Verde Museum Association). Let x be the weight of a fawn in kilograms. Convert each of the following x intervals to z intervals. 19 < x

**Solve:**

The z-score is calculated using the formula:

*z*=*x*−*μ/*​ *σ*

Where:

* *x* is the given value,
* *μ* is the mean of the distribution,
* *σ* is the standard deviation of the distribution, and
* *z* is the z-score.

Given that *μ*=27.2 kilograms and *σ*=4.3 kilograms

Calculating the z-scores for the lower and upper bounds of the interval:

For the lower bound, *x*=19:

19−27.2​/4.3

​≈−1.907

For the upper bound, *x*=∞ (since we're dealing with the right tail):

​=∞−27.2​/4.3

​ Since ∞∞ is not a finite value, we know that ​ will approach +∞.

Therefore, the z-interval corresponding to the weight interval 19<*x* is approximately −1.907<*z*<+∞.

Top of Form

**Sampling Distribution of the Mean, Point estimate, Confidence Intervals Problems**

1. (9e) Consider a random variable X that has a standard normal distribution with mean μ = 0 and standard deviation σ = 1. What value cuts off the lower 10% of the distribution of means?

**Solve:**

The 10th percentile of a standard normal distribution corresponds to a *z*-score such that 10% of the data lies below it.

Using a standard normal distribution table the *z*-score corresponding to the 10th percentile.

For a standard normal distribution, *z*0.10​ is approximately -1.28.

Therefore, the value that cuts off the lower 10% of the distribution of means in a standard normal distribution is approximately -1.28.

1. (14c) For the population of adult males in the United States, the distribution of weights is approximately normal with mean μ = 172.2 pounds and standard deviation σ = 29.8 pounds. What is the lower bound for 80% of the mean weights?

**Solve:**

To find the lower bound for 80% of the mean weights, we need to find the value of weight below which 80% of the mean weights lie.

Given that the distribution is approximately normal with a mean *μ*=172.2 pounds and a standard deviation *σ*=29.8 pounds, we need to find the *z*-score that corresponds to the 80th percentile of a standard normal distribution.

In other words, we're looking for the *z*-score, denoted as *z*0.80​, such that 80% of the data falls below it.

Using a standard normal distribution table, we find that *z*0.80​ is approximately 0.8416.

Now, we'll use the formula to convert this *z*-score to a weight value:

*z*=*x*−*μ/σ*​

Where:

* *x* is the weight value,
* *μ*=172.2 pounds is the mean weight,
* *σ*=29.8 pounds is the standard deviation, and
* *z*=0.8416 is the *z*-score corresponding to the 80th percentile.

We rearrange the formula to solve for *x*:

*x*=*μ*+*z*×*σ*

*x*=172.2+0.8416×29.8

≈172.2+25.08

≈197.28

So, the lower bound for 80% of the mean weights is approximately 197.28 pounds.

1. (11e) Of n1 randomly selected male smokers, X1 smoked filter cigarettes, whereas of n2 randomly selected female smokers, X2 smoked filter cigarettes. Let p1 and p2 denote the probabilities that a randomly selected male and female, respectively, smoke filter cigarettes. Estimate the standard error of your estimator if n1 = n2 = 200, x1 = 127, and x2 = 176.

**Solve:**

The standard error of the estimator for the difference in proportions can be estimated using the formula:

*SE*(*p*​1​−*p*​2​)=

Where:

* *p*1​ and *p*2​ are the sample proportions of male and female smokers who smoke filter cigarettes, respectively.
* *n*1​ and *n*2​ are the sample sizes of male and female smokers, respectively.

Given that *n*1​=*n*2​=200, *x*1​=127 (number of male smokers who smoke filter cigarettes), and *x*2​=176 (number of female smokers who smoke filter cigarettes), we can estimate *p*​1​ and *p*​2​ as:

*p*​1​= ​*x*1/ *n*1​​=127/200​=0.635

*p*​2​= ​*x*2/ *n*2​​=176/200​=0.88

Now, we can substitute these values into the formula to calculate the standard error:

*SE*(*p*​1​−*p*​2​)=

SE (*p*1​−*p*​2​)=0.001686875​

*SE*(*p*1​−*p*​2​)≈0.04106

Therefore, the estimated standard error of the estimator is approximately 0.041060.